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A MODEL AND ANALYSIS
OF THE EFFECTS OF ELECTRONIC WARFARE (EW)
IN ANTI-SURFACE WARFARE (ASUW)

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Consider the scenario of missile-carrying platforms attacking a set of targets. Electronic-warfare countermeasures can be used to attenuate and disperse the missile arrivals. The model considered here includes parameters to describe countermeasures involving decoys, attack coordination, jamming and acquisition, and missile accuracy. The mean and standard deviation of the missile arrival rate are calculated as a function of time. Summary measures for attenuation and dispersion are also calculated.

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1. INTRODUCTION

This report describes the analysis leading to modifications of a computer program that analyzes an electronic-warfare (EW) countermeasures model. The model, original analysis, and original computer program are discussed in Robert B. Washburn, Jr., "Dispersion Measure of Effectiveness of Electronic Warfare Combat Systems", technical report TR-999, ALPHATECH, Inc., Burlington, MA 01803, January 1985, which was prepared for Dr. Michael Melich, Naval Research Laboratory, Washington, D.C.

The next section discusses the context and some of the assumptions of the model. The third section defines the notation needed in the probabilistic analysis of the model in Section 4. The Appendix contains information for users of the computer program, which is implemented in *MICROSOFT BASIC* for the *APPLE MACINTOSH* computer.

2. THE MODEL

Consider a set of missile-carrying platforms preparing to attack a set of surface targets. The targets, which are stationary, are composed of some real and possibly some decoy targets, each with an associated probability of appearing to be real. Before the attack, a subset of the real and decoy targets are located and identified as real targets. These perceived real targets are allocated to the various attack platforms so as to distribute missiles evenly.

The plan is for all attack platforms to begin firing so that missiles begin to arrive at the targets simultaneously. However, two problems disperse the initial firing times of the attack platforms: location error and jamming. Consider some one attack platform. Action begins when the attack platform, approaching the target at a fixed speed, reaches the nominal launch range. The first problem is that the location of the target is not known precisely, which causes a positioning error that depends on the random actual location. The second problem is that the attack platform may be electronically jammed, preventing it from launching immediately. Jamming begins, with a specified probability, only at the nominal launch range. If jammed, the attack platform works to acquire the target electronically, at some point succeeding, if by no other means than burnthrough. Burnthrough occurs when the attack platform is so close to the target that jamming is irrelevant. When the target is acquired, either at the nominal launch range or after, the missiles are launched over a fixed duration.

There are possibly several types of attack platforms, each with a specified number of platforms of that type, missiles per platform, duration of fire, missile velocity, platform speed, nominal launch range, burnthrough range, probability of jamming, acquisition rate, and probability of each missile hitting a target.

Additional model properties, in the form of assumptions, are stated as needed in the analysis of Section 4.

3. NOTATION

The notation, summarized here both for ease of reference and as an overview of the components of the model, follows the convention that random variables are upper case and their realizations and other constants are lower case.

3.1. Random Variables

R: The number of real targets thought to be real by the attacker.

D: The number of decoy targets thought to be real by the attacker.

X: The number of platforms attacked. $(X = R + D)$

$T_{i,j}^{(0)}$: The time perturbation for the j^{th} attack platform of type i due to target-location error.

$T_{i,j}^{(1)}$: The time delay for the j^{th} attacking platform of type i to acquire the target after jamming.

$T_{i,j}^{(2)}$: The time delay for the j^{th} attacking platform of type i to burnthrough.

$T_{i,j}$: The time missiles from the j^{th} attack platform of type i begin to arrive at the target, where $t = 0$ is the planned time.

$H_{i,j,k}$: An indicator variable equal to one if the k^{th} missile from the j^{th} attack platform of type i hits a target and zero if it misses.

$I(t)_{[F,G]}$: An indicator random variable equal to one if $F \leqslant t \leqslant G$ and zero otherwise, for specified random variables F and G.

$Y_{i,j}(t)$: At time t, the random rate per minute at which missiles from the j^{th} attack platform of type i arrive at each of the real targets when EW countermeasures are active.

$Y(t)$: At time t, the random rate per minute at which missiles arrive at each of the real targets when EW countermeasures are active.

3.2. Measures of Effectiveness

$\{y_{IS}(t): -\infty < t < \infty\}$: The deterministic rate per minute at which missiles hit each real target at time t under an *ideal saturation attack*, which corresponds to EW countermeasures, as a function of time.

$\{\mathrm{E}(Y(t)): -\infty < t < \infty\}$: The expected value of $Y(t)$ as a function of time.

$\{\mathrm{V}(Y(t)): -\infty < t < \infty\}$: The variance of $Y(t)$ as a function of time.

$\mathrm{E}(X^{-1} \mid R > 0)$: The expected value of the reciprocal of the number of identified targets, given that at least one real target is identified.

$\{p_{i,j}(t): -\infty < t < \infty\}$: The probability the j^{th} attack platform of type i is firing as a function of time.

α: Attenuation. The ratio of the number of missile hits per real target with EW countermeasures to that corresponding to an ideal saturation attack. Attenuation is a summary number that is a function of the first two measures of effectiveness.

β: Dispersion. The ratio of variances of the time of randomly selected missile hits with and without EW countermeasures. Dispersion is a summary number calculated from the first two measures of effectiveness.

3.3. Global Parameters

n_r: Number of real targets. $(1 \leqslant n_r)$

n_d: Number of decoy targets. $(0 \leqslant n_d)$

p_r: Probability that a real target is thought to be real. $(0 \leqslant p_r \leqslant 1)$

p_d: Probability that a decoy target is thought to be real. $(0 \leqslant p_d \leqslant 1)$

d: Maximum target-location error, in nautical miles. $(d > 0)$

n: Number of attack platform types. $(n \geqslant 1)$

3.4. Attack Platform Parameters $(i = 1, 2, \cdots, n)$

p_i: Number of platforms of type i. $(0 \leqslant p_i)$

m_i: Number of missiles per platform of type i. $(0 \leqslant m_i)$

f_i: Time for a platform of type i to fire all its missiles. $(0 < f_i)$

v_i: Velocity of missiles launched from platforms of type i in machs. $(0 < v_i)$

s_i: Speed of attack platforms of type i in knots. $(0 < s_i)$

l_i: Nominal launch range in nautical miles. $(0 \leqslant l_i)$

b_i: Burnthrough range in nautical miles. $(0 \leqslant b_i)$

j_i: Probability that attack platforms of type i are jammed. $(0 \leqslant j_i \leqslant 1)$

a_i: Rate at which jammed attack platforms of type i acquire the target, in reciprocal minutes. $(0 \leqslant a_i)$

h_i: Probability that a missile launched from an attack platform of type i hits its intended target. $(0 \leqslant h_i \leqslant 1)$

Be careful. The notations p_r, p_d, p_i and $p_{i,j}(t)$ look similar, but have quite different meanings.

4. THE ANALYSIS

From the model we have

$$Y(t) = \sum_{i=1}^{n} \sum_{j=1}^{p_i} Y_{i,j}(t) = \sum_{i=1}^{n} \sum_{j=1}^{p_i} \left[\frac{\sum_{k=1}^{m_i} H_{i,j,k}}{X f_i} \right] I(t)_{[T_{i,j},\, T_{i,j}+f_i]}$$

where

$$T_{i,j} = T_{i,j}^{(0)} + \min(T_{i,j}^{(1)},\, T_{i,j}^{(2)})$$

The entire analysis is to determine various properties of $Y(t)$, the random missile hit rate per identified real target. (The rate for unidentified real targets is zero.)

Let us emphasize: The point of view of the analysis is that of any one of the real targets being attacked. This perspective sometimes differs from that of the system of targets, especially with respect to the effectiveness of undetected real targets. From the system point of view, undetected real targets are good. From the point of view taken in this analysis, undetected real targets are bad, because the missile hit rate on the target of interest, which is assumed detected, is increased.

In addition to the analysis point of view, we need to emphasize another point — the meaning of *rate*. In Section 2.1, $Y(t)$ was defined as the "random rate per minute at which missiles arrive at each of the real targets at time t." Now having defined $Y(t)$ mathematically we can discuss its meaning more precisely.

The rate $Y_{i,j}(t)$ for platform j of type i is obtained by dividing the number of missile hits by the duration of firing, f_i, and the number of targets, X, for all t while the platform is firing. $Y_{i,j}(t)$ is zero for all t outside the duration of firing. Thus, $Y(t)$ depends upon action at times other than only t. Therefore the definition of $Y(t)$ has already made an assumption about the timing of missile firings within the firing duration. The level rate implies

Assumption 1. *Every time $t \in [T_{i,j}, T_{i,j} + f_i]$ has the same likelihood of seeing a missile from platform j of type i hit a target.*

The firing protocol is not specified, but is restricted to those protocols that spread the threat evenly over the firing duration. The analysis does not yield peaks at the beginning and end of the firing duration, which would be expected with the assumption of firing one missile every $f_i/(m_i - 1)$ minutes, with the first at the very beginning of the firing duration and the last exactly f_i minutes later. Two firing protocols that are consistent with the definition are to fire one missile at a time uniformly distributed over each interval of length f_i/m_i and to fire m_i missiles at times uniformly distributed over the firing duration. These two protocols would then require the the probability of mis-

sile i,j,k hitting a target be constant over all missiles k.

4.1. The Ideal Saturation Attack Hit Rate $y_{IS}(t)$

As an absolute comparison, the analysis uses an ideal saturation attack to represent no EW countermeasures. The ideal saturation attack is achieved by having all real targets recognized as such, having no decoys thought to be real, having no location error, having no attackers jammed, and having every missile strike a target. That is, $p_r = 1$, $p_d = 0$, $d = 0$, $j_i = 0$, and $h_i = 1$ in the definition of $Y(t)$. Therefore,

$$y_{IS}(t) = \sum_{i=1}^{n} \sum_{j=1}^{p_i} \left[\frac{\sum_{k=1}^{m_i} 1}{n_r f_i} \right] I(t)_{[0, f_i]} = n_r^{-1} \sum_{i=1}^{n} (p_i m_i / f_i)\, I(t)_{[0, f_i]}$$

4.2. Expected Value of $Y(t)$

Consider the expected number of missiles per minute, $E(Y(t))$, under the specified parameter values describing the EW countermeasures. Condition on the number of targets, X, to obtain

$$E(Y(t)) = \sum_{x=1}^{n_r + n_d} \sum_{i=1}^{n} \sum_{j=1}^{p_i} E\left\{ \left[\frac{\sum_{k=1}^{m_i} H_{i,j,k}}{x f_i} \right] I(t)_{[T_{i,j},\, T_{i,j} + f_i]} \;\middle|\; X = x \right\} P(X=x \mid R > 0)$$

The condition that $R > 0$ reflects the modeling perspective that $Y(t)$ corresponds to a real target, so the case in which only decoys are fired upon is not considered.* Now assume

Assumption 2. *The $H_{i,j,k}$'s are independent of $T_{i,j}$; i.e., whether a missile hits or not is independent of the time perturbations.*

Assumption 3. *The $H_{i,j,k}$'s and $I(t)$ are independent of X; i.e., whether a missile hits or not and the time delays are independent of the number of target platforms attacked.*

Then

$$E(Y(t)) = \left[\sum_{i=1}^{n} \sum_{j=1}^{p_i} \frac{E(I(t)_{[T_{i,j},\, T_{i,j} + f_i]}) \sum_{k=1}^{m_i} E(H_{i,j,k})}{f_i} \right] \left[\sum_{x=1}^{n_r + n_d} x^{-1} P(X=x \mid R > 0) \right]$$

* The probability that only decoys are attacked is easy to compute under the independence assumptions made in this analysis.

Now assume

Assumption 4. *The hit probability of the k^{th} missile launched from the j^{th} attack platform of type i is constant for all k and for all j.*

Then

$$E(Y(t)) = \left[\sum_{i=1}^{n} P(T_{i,j} \leq t \leq T_{i,j} + f_i) \frac{p_i\, m_i\, h_i}{f_i} \right] E(X^{-1} \mid R > 0)$$

Therefore, only two quantities not specified directly as parameters must be calculated: the probability that an arbitrary attack platform of type i is firing at time t and the expected value of the reciprocal of the number of identified targets conditional on there being at least one real target identified. These are the topics of the next two subsections.

4.2.1. Expected Value of X^{-1} Given $R > 0$

Consider the expected value first, since it is straightforward. Condition on X, the number of targets, and R, the number of real targets identified as such.

$$E(X^{-1} \mid R > 0) = \sum_{x=1}^{n_r + n_d} x^{-1} \sum_{r=1}^{n_r} P(D = x - r \mid R = r)\, P(R = r \mid R > 0)$$

Now assume

Assumption 5. *Each real target is identified as such with equal probability.*

Assumption 6. *Each decoy is identified as a real target with equal probability.*

Assumption 7. *Each decoy and each real target is identified independently of the others.*

Then both R and $D = X - R$ are binomial random variables and

$$E(X^{-1} \mid R > 0) = \sum_{x=1}^{n_r + n_d} x^{-1} \sum_{r=\max(1,\, x-n_d)}^{\min(x,\, n_r)} \left[\binom{n_d}{x-r} p_d^{x-r} (1-p_d)^{n_d-(x-r)} \right] \left[\frac{\binom{n_r}{r} p_r^r (1-p_r)^{n_r - r}}{1 - (1-p_r)^{n_r}} \right]$$

The denominator, which is the probability of at least one real target being attacked, is dependent upon neither x nor r and therefore is calculated only once.

4.2.2. Probability of Firing at Time t

Now consider the other quantity necessary to calculate $E(Y(t))$: the probability that an attack platform of type i is firing at time t. For an arbitrary i and j, first reorganize the inequality and then condition on whether the attack platform is initially

jammed.

$$p_{i,j}(t) \equiv P(T_{i,j} \leq t \leq T_{i,j} + f_i)$$
$$= P(t - f_i \leq T_{i,j} \leq t)$$
$$= [\,(1 - j_i)\ P(t - f_i \leq T^{(0)}_{i,j} \leq t)]$$
$$+\ [\,j_i\ P(t - f_i \leq T^{(0)}_{i,j} + \min(T^{(1)}_{i,j},\ T^{(2)}_{i,j}) \leq t)]$$

Now assume

Assumption 8. *$T^{(0)}_{i,j}$ is uniformly distributed between $-d/v_i$ and d/v_i.*

Assumption 9. *$T^{(1)}_{i,j}$ is exponentially distributed with rate a_i.*

Assumption 10. *$T^{(2)}_{i,j}$ is the constant $t^{(2)}_{i,j} = \max(0, (l_i - b_i) / s_i)$.*

Assumption 11. *$T^{(0)}_{i,j}$ and $T^{(1)}_{i,j}$ are independent; i.e., the location-error perturbation and the jamming-acquisition delay are independent.*

Then

$$p_{i,j}(t) = (1 - j_i)\left[\left(\frac{v_i}{2\ d}\right)\phi\Big(\,(t - f_i,\ t),\ (-d/v_i,\ d/v_i)\Big)\right]$$
$$+\ j_i\ P(t - f_i \leq T^{(0)}_{i,j} + T^{(1)}_{i,j} \leq t)\ P(T^{(1)}_{i,j} \leq T^{(2)}_{i,j})$$
$$+\ j_i\ P(t - f_i \leq T^{(0)}_{i,j} + T^{(2)}_{i,j} \leq t)\ P(T^{(2)}_{i,j} \leq T^{(1)}_{i,j})$$

where ϕ is the function that returns the length of the the intersection of the two intervals and

$$P(T^{(1)}_{i,j} \leq T^{(2)}_{i,j}) = P(T^{(1)}_{i,j} \leq t^{(2)}_{i,j}) = 1 - \exp(-a_i\ t^{(2)}_{i,j})$$

since $t^{(2)}_{i,j}$ is nonnegative by definition. The probability that burnthrough occurs before acquisition is the complementary probability

$$P(T^{(1)}_{i,j} > t^{(2)}_{i,j}) = \exp(-a_i t^{(2)}_{i,j})$$

Since $T^{(2)}_{i,j}$ is a constant,

$$P(t - f_i \leq T^{(0)}_{i,j} + T^{(2)}_{i,j} \leq t) = P(t - t^{(2)}_{i,j} - f_i \leq T^{(0)}_{i,j} \leq t - t^{(2)}_{i,j})$$
$$= \left[\left(\frac{v_i}{2\ d}\right)\phi\Big(\,(t - t^{(2)}_{i,j} - f_i,\ t - t^{(2)}_{i,j}),\ (-d/v_i,\ d/v_i)\Big)\right]$$

Finally we must consider the most troublesome quantity: the probability that platform j of type i is firing at time t conditional upon jamming and that the target is acquired before burnthrough. The difficulty is that now the convolution of $T_{i,j}^{(0)}$ and $T_{i,j}^{(1)}$ must be considered. As usual, begin by conditioning, this time on $T_{i,j}^{(1)}$.

$$P(t-f_i \leq T_{i,j}^{(0)} + T_{i,j}^{(1)} \leq t) = P(t-T_{i,j}^{(1)}-f_i \leq T_{i,j}^{(0)} \leq t-T_{i,j}^{(1)})$$

$$= \int_0^{t_{i,j}^{(2)}} P(t-t_{i,j}^{(1)}-f_i \leq T_{i,j}^{(0)} \leq t-t_{i,j}^{(1)}) \left[\frac{a_i \exp(-a_i\, t_{i,j}^{(1)})}{1-\exp(-a_i\, t_{i,j}^{(2)})}\right] dt_{i,j}^{(1)}$$

where the conditional density function of $T_{i,j}^{(1)}$ is relevant, since now $T_{i,j}^{(1)} \leq t_{i,j}^{(2)}$. The integrand is now easy to handle, since it is of the same form as handled by the ϕ function twice earlier. Therefore,

$$P(t-f_i \leq T_{i,j}^{(0)} + T_{i,j}^{(1)} \leq t) = \frac{(v_i\,/\,(2d)) \int_0^{t_{i,j}^{(2)}} \phi\Big((t-t_{i,j}^{(1)}-f_i,\ t-t_{i,j}^{(1)}),\ (-d/v_i,\ d/v_i)\Big)\ a_i \exp(-a_i\, t_{i,j}^{(1)})\ dt_{i,j}^{(1)}}{1-\exp(-a_i\, t_{i,j}^{(2)})}$$

The denominator cancels with $P(T_{i,j}^{(1)} \leq t_{i,j}^{(2)})$. The remaining problem is evaluation of the integral.

Because of the complicated nature of the overlap function ϕ, numerical solution is tempting. However, since the analysis must be performed for many time epochs t, and since the analysis is to be performed interactively on a microcomputer, a closed-formed solution is worth the effort. Another advantage is that the accuracy does not depend upon the user's choice of plotting parameters.

Once again, the approach is to break the problem into manageable pieces. Consider the nature of $\phi((a,b),(c,d))$, where for a moment a, b, c, and d have no meaning other than to define two arbitrary intervals. Two intervals have one of six possible relationships. Two are trivial — one interval lying above another — resulting in $\phi((a,b),(c,d)) = 0$. The four interesting cases are $a < c < b < d$, $a < c < d < b$, $c < a < b < d$, and $c < a < d < b$. Since the six relationships are mutually exclusive, the integral is the sum of six other integrals (two with zero values) with ϕ replaced by the appropriate overlapping length. As $t_{i,j}^{(1)}$ goes from 0 to $t_{i,j}^{(2)}$, different integrals become relevant. The subprogram *integrator* in the appendix implements the various formulas.

4.3. Variance of $Y(t)$

Given $p_{i,j}(t)$ and $E(Y(t))$ from the last section, the calculation of $V(Y(t))$ is straightforward under

Assumption 12. *Each attack platform is independent of all other attack platforms; i.e., every random variable not sharing i,j subscripts are mutually independent.*

Then

$$V(Y(t)) = \sum_{i=1}^{n} \sum_{j=1}^{p_i} V(Y_{i,j}(t)) = \sum_{i=1}^{n} p_i \, V(Y_{i,j}(t))$$

where the second equality is true since all attack platforms of a single type are identically distributed by definition of *type*.

Now $V(Y_{i,j}(t)) = E(Y_{i,j}^2(t)) - E_{i,j}^2(Y(t))$. As a subargument from Section 3.2 we have

$$E(Y_{i,j}(t)) = (m_i \, h_i \,/\, f_i) \; p_{i,j}(t) \; E(X^{-1} \mid R > 0)$$

so we concentrate on

$$E(Y_{i,j}^2(t)) = E\left[\left(\left[\sum_{k=1}^{m_i} H_{i,j,k} \,/\, (X f_i)\right] \; I(t)_{[T_{i,j},\, T_{i,j} - f_i]} \right)^2\right]$$

Taking the square, conditioning on X, and invoking the earlier assumption that $H_{i,j,k}$ and $I(t)$ are independent of X, we obtain

$$E(Y_{i,j}^2(t)) = \sum_{x=1}^{n_r + n_d} E\left[\left(\sum_{k=1}^{m_i} H_{i,j,k} \right)^2 \; I^2(t)_{[T_{i,j},\, T_{i,j} + f_i]} \,/\, f_i^2 \right] \; x^{-2} \, P(X = x \mid R > 0)$$

Then using $I^2(t) = I(t)$, invoking the assumption of independence between $H_{i,j,k}$ and $T_{i,j}$, taking the square, and moving the summation to the right, we have

$$E(Y_{i,j}^2(t)) = f_i^{-2} \; E\left[\left(\sum_{k=1}^{m_i} \sum_{l=1}^{m_i} H_{i,j,k} \, H_{i,j,l} \right)\right] \; p_{i,j}(t) \; E(X^{-2} \mid R > 0)$$

Now assume

Assumption 14. *All missiles fired by attack platform j of type i are independent; i.e., for all i and j, $H_{i,j,k}$ is independent of $H_{i,j,l}$ if $k \neq l$.*

Then noting that $H_{i,j,k}^2 = H_{i,j,k}$ yields

$$E(Y_{i,j}^2(t)) = f_i^{-2} \, [m_i \, h_i + m_i(m_i - 1) h_i^2] \; p_{i,j}(t) \; E(X^{-2} \mid R > 0)$$

$E(X^{-2} \mid R > 0)$ can be calculated essentially for free when $E(X^{-1} \mid R > 0)$ is calculated, by substituting x^{-2} for x^{-1} in Section 4.2.1.

4.4. Attenuation

Attenuation, denoted by α, is a scalar measure of the effectiveness of the EW countermeasures. The measure is one of comparison to an ideal saturation attack: every real target is identified as such, no decoys are mistaken for real targets, no attackers are

jammed, no attacker has any location error, and every missile hits. By definition,

$$\alpha \equiv \frac{\int_{-\infty}^{\infty} \mathrm{E}(Y(t))\, dt}{\int_{-\infty}^{\infty} y_{IS}(t)\, dt}$$

Graphically, α is the ratio of the areas under the two rate curves arising from the two scenarios. While α can be calculated by numerically integrating the two rates, a closed-form expression can be obtained by noting that each integral is the expected value of the number of missiles that hit each real target, without regard to time. That is,

$$\alpha = \frac{\mathrm{E}\left(\sum_{i=1}^{n} \sum_{j=1}^{p_i} \sum_{k=1}^{m_i} H_{i,j,k} \,/\, X \right)}{\sum_{i=1}^{n} p_i\, m_i \,/\, n_r}$$

Using logic similar to that used earlier and no new assumptions, we have

$$\alpha = \frac{\left(\sum_{i=1}^{n} p_i m_i h_i \right) \mathrm{E}(X^{-1} \mid R > 0)}{\left(\sum_{i=1}^{n} p_i m_i \right) / n_r}$$

For $n = 1$, attenuation is a function of h_i, n_r, n_d, p_r, and p_d. For $n > 1$, attenuation is also a function of $p_i m_i$.

Good EW countermeasures have low attenuation. However, keep in mind the point of view of the analysis — that of a real target being fired upon. If there are many real targets, the ideal saturation attack will fire at all of them, diluting the rate for any one real target.. Countermeasures could hide some real targets, thereby increasing the rate for those fired upon. For this reason, attenuation can be greater than one.

4.5. Dispersion

The dispersion measure, denoted by β, is another scalar measure of the effectiveness of the EW countermeasures. Again it is a comparison to an ideal saturation attack, but

now the criterion is the dispersion around the average time of missile hits. In particular,

$$\beta = \frac{\int_{-\infty}^{\infty} \mathrm{E}(Y(t))\, t^2\, dt - \left[\int_{-\infty}^{\infty} \mathrm{E}(Y(t))\, t\, dt\right]^2}{\int_{-\infty}^{\infty} y_{IS}(t)\, t^2\, dt - \left[\int_{-\infty}^{\infty} y_{IS}(t)\, t\, dt\right]^2}$$

Good EW countermeasures have high dispersion.

A closed-form solution for β would be useful, since numerical calculation is slow and prone to error via both time truncation and interpolation.

5. APPENDIX

This appendix is a short discussion of the computer implementation of the model and analysis discussed in the body of this report. The computer program runs interactively on the *APPLE MACINTOSH* microcomputer. The mechanics of running the program are straightforward if the model of Section 2 is understood. If you have not done so, read the introduction of Section 2.

The current version is a modification of the original ALPHATECH program. The input/output routines are essentially the same; the analysis has been mostly rewritten.

The program was tested on three *MACINTOSH* computers: One with *MACBOTTOM*, one with *HYPERDRIVE*, and another with no hard disk. Because the modified program sometimes writes directly to the printer rather than dumping the screen, some code that works for one system won't work for another. If you have difficulty printing, try to use the system without the hard disk; usually this can be done by booting the system with the program disk in the machine.

The program consists of four types of screens, two for specifying model parameters and two for displaying analysis results. The second screen is for specifying parameters for each type of attack platform. The first screen is for specifying everything else — target information, maximum target location error, number of attack-platform types and plotting parameters. Other than the plotting parameters, Section 3.3 defines the global parameters and Section 3.4 defines the attack-platform parameters in the order they are to be specified on the screen.

There are two output screens. Both center on axes showing time horizontally and hits per minute vertically. The first shows the rate at which a real target is fired upon as a function of time, under the assumption of an ideal saturation attack. The second begins by recreating the first output screen and adding other functions of time and two scalar measures of effectiveness, as discussed in Sections 3.2 and 4. The two heavy lines are the expected rate per minute of missiles hitting a real target under the two scenarios of EW countermeasures and of an ideal saturation attack. In addition, two lighter lines are plotted. These lines are one standard deviation above and below the EW mean rate function. They provide an idea of the variability from the mean. Note that deviations above the upper light line are not uncommon.

The second output screen is obtained by pressing carriage return while looking at the first output screen. The time in seconds to calculate the values for the second screen is roughly the product of the number of time points plotted and the number of attack-platform types. If the cursor is blinking, then the program is waiting for input; otherwise, the long wait is due to computation.

The two scalar measures of the effectiveness of the electronic-warfare countermeasures are attenuation and dispersion, discussed in Sections 4.4 and 4.5, respectively. Attenuation is calculated analytically and dispersion is calculated by numerical integration. The plotting parameters are important to the calculation of dispersion, in that if the time interval considered is too short or the time between plotted points is too long, the calculated dispersion will be incorrect. To provide the user with a warning when the plotting parameters are inadequate, the attenuation is calculated both numerically and analytically. If the answers differ by more than 5%, then a warning is printed.

Any of the four screens can be dumped to the printer. However, printing only the fourth screen is usually sufficient, since it contains all the information of the third screen and the program also prints all model parameters whenever the fourth screen is printed.

An example follows. A listing of the *BASIC* program is attached.

6. ACKNOWLEDGMENT:

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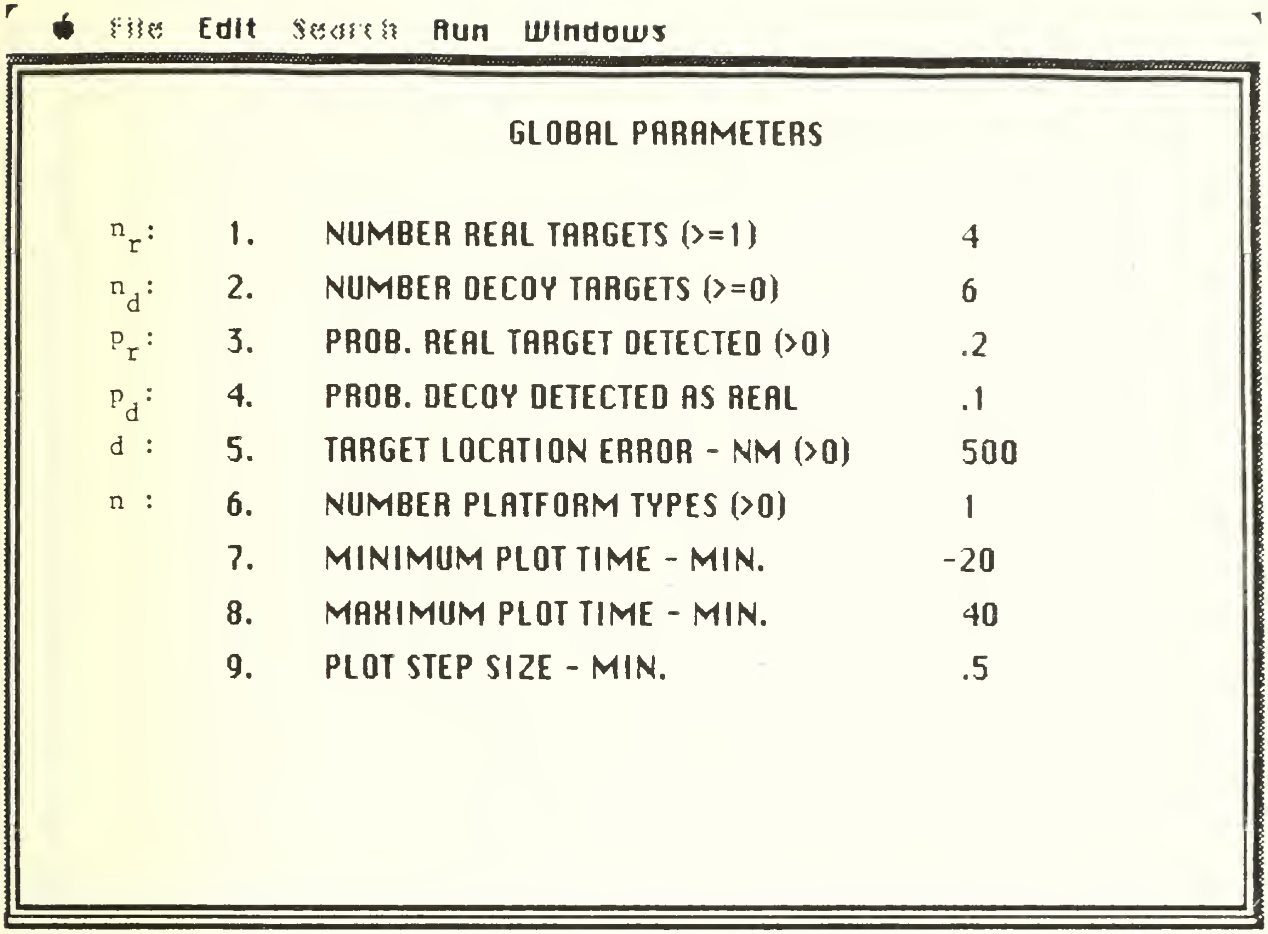


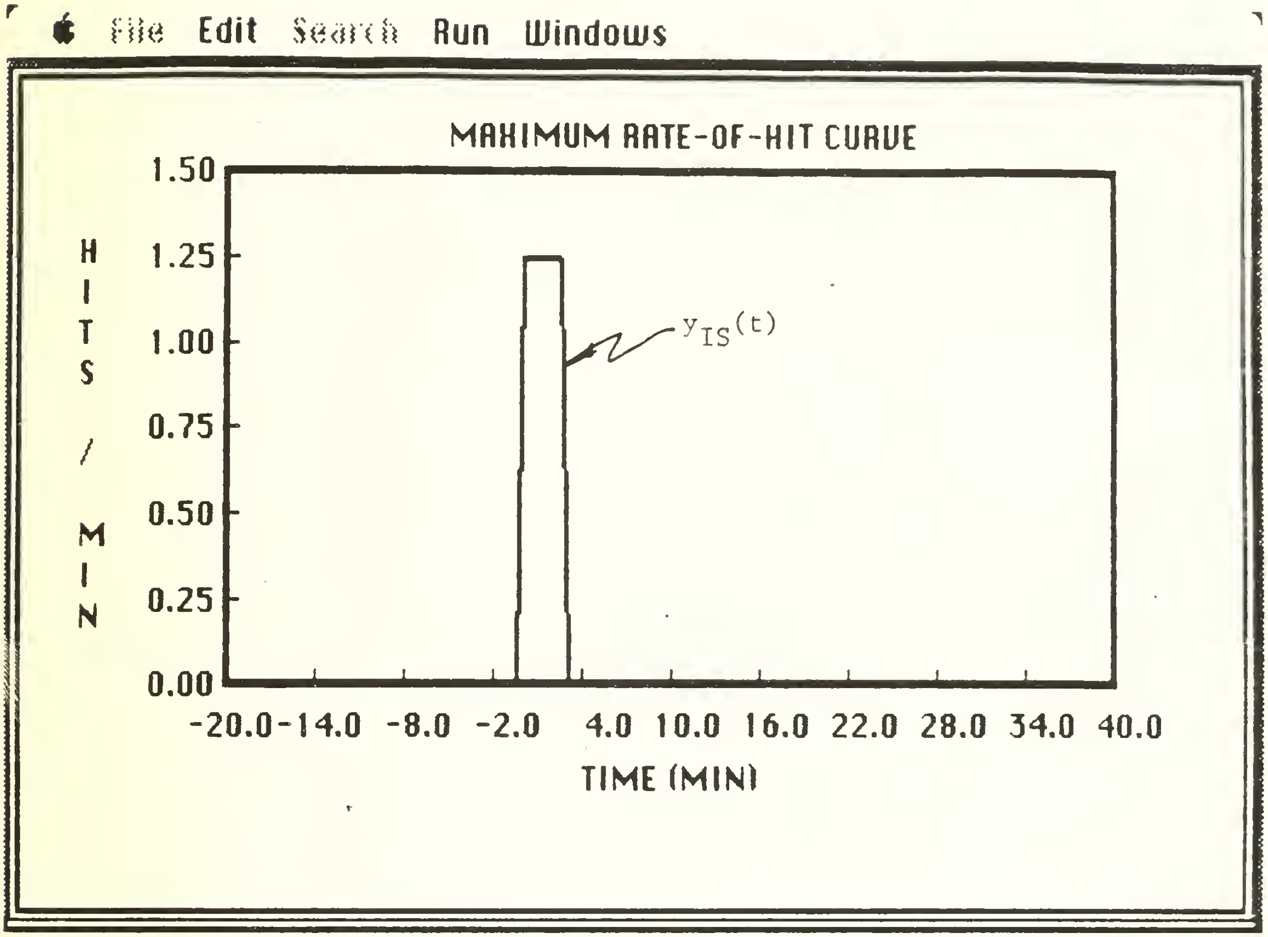
Figure 1. Screen for Specifying Global Parameters

File Edit Search Run Windows

PLATFORM PARAMETERS 1

p_i:	1.	NUMBER PLATFORMS (>=0)	3
m_i:	2.	NUMBER MISSILES PER PLATFORM (>=0)	5
f_i:	3.	TIME TO FIRE ALL MISSILES - MIN(>0)	3
v_i:	4.	MISSILE SPEED - MACH (>0)	4
s_i:	5.	PLATFORM SPEED - KTS (>=0)	300
l_i:	6.	NOMINAL LAUNCH RANGE - NM (>=0)	200
b_i:	7.	BURNTHROUGH RANGE - NM (>=0)	50
j_i:	8.	PROB. ACQUISITION RADAR JAMMED	.4
a_i:	9.	RATE OF ACQUISITION - 1/MIN. (>=0)	.2
h_i:	10.	PROB. LAUNCHED MISSILE HITS	.9

Figure 2. Screen for Specifying Attack Platform Parameters



Screen 3. First Output Screen, Containing the Hit Rate for the Ideal-Saturation Attack

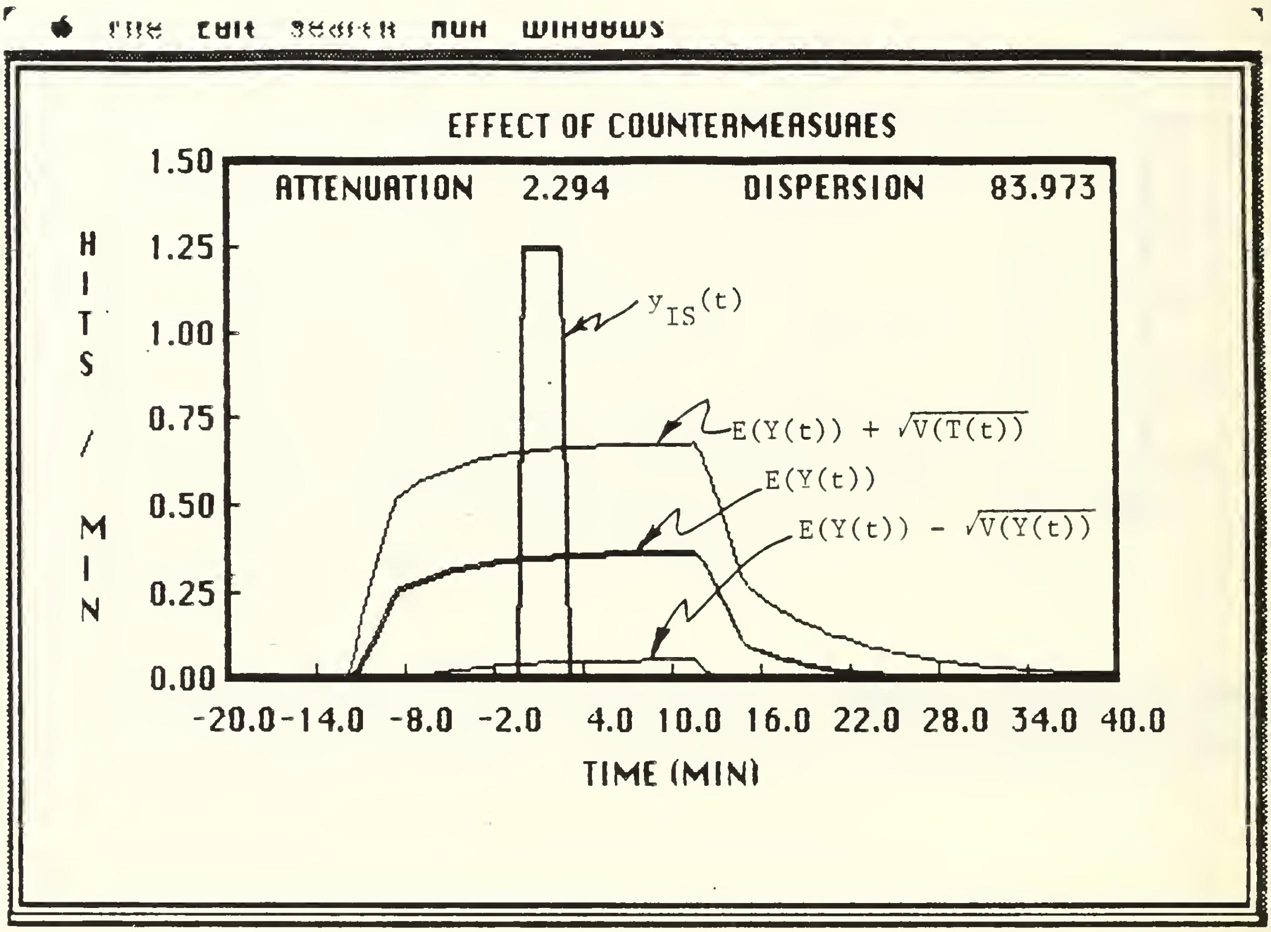


Figure 4. Second Output Screen, Containing all Analysis Results, Plus the Printed Model Parameters.

GLOBAL PARAMETERS	
NUMBER REAL TARGETS (>=1)	4
NUMBER DECOY TARGETS (>=0)	6
PROB. REAL TARGET DETECTED (>0)	.2
PROB. DECOY DETECTED AS REAL	.1
TARGET LOCATION ERROR - NM (>0)	500
NUMBER PLATFORM TYPES (>0)	1
MINIMUM PLOT TIME - MIN.	-20
MAXIMUM PLOT TIME - MIN.	40
PLOT STEP SIZE - MIN.	.5
TYPE 1 PLATFORM PARAMETERS	
NUMBER PLATFORMS (>=0)	3
NUMBER MISSILES PER PLATFORM (>=0)	5
TIME TO FIRE ALL MISSILES - MIN(>0)	3
MISSILE SPEED - MACH (>0)	4
PLATFORM SPEED - KTS (>=0)	300
NOMINAL LAUNCH RANGE - NM (>=0)	200
BURNTHROUGH RANGE - NM (>=0)	50
PROB. ACQUISITION RADAR JAMMED	.4
RATE OF ACQUISITION - 1/MIN. (>=0)	.2
PROB. LAUNCHED MISSILE HITS	.9

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riginal, January 1985, Robert Washburn, Alphatech, Inc.
odified, March 1986, Bruce Schmeiser, Purdue University.

Figure 5. Model Parameters as Printed with the Second Output Screen.

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